

# **Argon + Carbon Dioxide Gaseous Mixture Viscosities and Anisotropic Pair Potential Energy Functions<sup>1</sup>**

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The viscosities of pure gaseous carbon dioxide and argon + carbon dioxide mixtures have been measured with a capillary flow viscometer. The viscosities are relative to those of argon, in the temperature range 213 to 353 K, and considered accurate to  $\pm 0.7\%$ . The pure-component viscosities agree closely with previous measurements. The mixture viscosities are used to calculate interaction viscosities and binary diffusion coefficients, which are compared with previous measurements. Interaction viscosities have been calculated, by use of the Mason–Monchick approximation, from the anisotropic pair potential energy functions for the unlike interaction proposed by Pack and his co-workers and by Hough and Howard. Comparison of these calculated interaction viscosities with those derived from our experiments and the higher-temperature measurements of Hobley, Matthews, and Townsend proves to be a powerful discriminant for the proposed anisotropic potential functions.

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**KEY WORDS:** argon; carbon dioxide; mixtures; pair potential energy functions; viscosity.

## **1. INTRODUCTION**

The transport properties of gases, in particular their viscosities, continue to be an important source of information in the study of intermolecular forces. Much attention has been focused on anisotropic interactions between atoms and molecules, such as the atom–quadrupole interaction of argon

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<sup>1</sup> Paper dedicated to Professor Joseph Kestin.

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and carbon dioxide studied in this work. Several angle-dependent anisotropic pair potential energy functions have been proposed for the Ar + CO<sub>2</sub> system, most notably by Pack and co-workers [1-3] and Hough and Howard [4, 5]. Howard and co-workers have also studied interactions between atoms and dipolar molecules, such as Ar + HCl and Xe + HCl [6, 7], as well as the Ar + OCS and N<sub>2</sub> + CO<sub>2</sub> interactions [8, 9].

The intermolecular interaction between two spherical molecules (or atoms) in the gas phase can be characterized in terms of a pair potential energy function  $U(r)$  which describes the overall resultant (positive) repulsive energy between the molecules at small separations  $r$ , the resultant (negative) attractive energy at larger distances, and the asymptotic approach of the resultant attractive energy to zero at very large separations. Two characteristic features of this function are, first, the separation at which the interaction energy passes through zero, this distance being referred to as the collision diameter  $\sigma$ , and, second, the position of minimum interaction energy  $U$ , i.e. of greatest attraction. This position is referred to as the equilibrium separation  $r_m$  at which the interaction energy has a value  $-\varepsilon$ , where  $\varepsilon$  is termed the well depth. If  $U(r)$  is known, the various transport properties of the system may be calculated by rigorous kinetic theory, which uses equations involving collision integrals,  $\Omega^{(1,s)}$ , which are integrals over the full range of energies, trajectories of collision, and orientations of the molecules during collisions. For a function  $U_{12}(r)$  which applies to the cross interaction of two unlike molecules, the properties which are calculated are hypothetical ones arising only from collisions between unlike molecules, while real experimental properties arise from a combination of both like and unlike interactions. The hypothetical viscosity of this type is known as the interaction viscosity  $\eta_{12}$ , which to first order in the Chapman-Enskog formulation, is

$$\eta_{12} = \frac{5}{16} \left( \frac{m_{12} k T}{\pi} \right)^{1/2} \frac{1}{\sigma_{12}^2 \Omega_{12}^{(2,2)*}(T_{12}^*)} \quad (1)$$

Here  $k$  is the Boltzmann constant,  $T$  is the thermodynamic temperature, and the molar mass  $m_{12}$  of the hypothetical species is twice the reduced mass of the system, i.e.,  $2m_1 m_2 / (m_1 + m_2)$ . The reduced collision integral  $\Omega_{12}^{(2,2)*}$  is a function of the reduced temperature  $T_{12}^* = kT/\varepsilon_{12}$ , where  $\varepsilon_{12}$  is the well depth of the unlike interaction.

In the case of mixtures of gaseous atoms or spherical molecules, the interaction viscosities can be "inverted" to yield an isotropic pair potential energy function [10]. For polyatomic systems, an effective isotropic potential function can be derived by inverting interaction viscosities, but its physical meaning is not clearly defined. Nevertheless, the forward calcula-

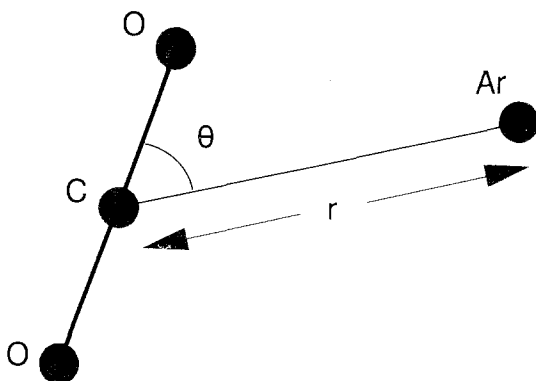


Fig. 1. The coordinates  $r$  and  $\theta$  for the interaction of rigid, linear CO<sub>2</sub> with Ar.

tion, from anisotropic potential energy surfaces  $U(r, \theta)$  to interaction viscosities, is useful. Most commonly the Mason–Monchick approximation [10], which is the classical equivalent of the “infinite order sudden” approximation, is employed. Within this approximation, the collision integrals  $\Omega_{12}^{(1,s)*}$  are defined as equally weighted averages for each fixed orientation  $\theta$ , so that

$$\Omega_{12}^{(1,s)*}(T^*) = \langle \Omega_{12}^{(1,s)*}(T^*, \theta) \rangle = \frac{1}{2} \int_{-1}^{+1} \Omega_{12}^{(1,s)*}(T^*, \theta) d(\cos \theta) \quad (2)$$

In this work, we assume that the carbon dioxide molecule is a rigid, linear rotor, and  $\theta$  and  $r$  are defined as shown in Fig. 1. It is also possible to calculate the interaction viscosities  $\eta_{12}$  of a system from the experimental measurements. The viscosities of both the pure components and at least one mixture are required. However, the calculation demands an estimate of  $\varepsilon_{12}$  to generate the ratio of reduced collision integrals  $A_{12}^* = \Omega_{12}^{(1,2)*} / \Omega_{12}^{(1,1)*}$  [10]. Fortunately,  $A_{12}^*$  is insensitive to the shape of the potential function used to generate it.

## 2. EXPERIMENTS

Argon + carbon dioxide viscosities have been measured with two capillary flow gas viscometers. The pressure drop of the gas, and hence its rate of flow, is measured as it moves from a sample vessel, through a coiled capillary tube mounted in a cryostat, and then into a “back” vessel at a lower pressure. The two sets of apparatus have been described fully in previous publications [11, 12].

Both the argon and the carbon dioxide were 99.998% pure, with

major impurity nitrogen. Experiments were carried out on mixtures of mole fractions 0.25, 0.50, and 0.75, measured gravimetrically to within 0.1% mole fraction of the component of lowest composition in a particular mixture. The measurements were made at six temperatures in the temperature range 213 to 353 K, measured with a platinum resistance thermometer calibrated to an accuracy of 0.1 K. Various pressure regimes were used, absolute pressures being measured by means of a cathetometer accurate to 0.05 Torr. The experiments at 213 K were repeated on the more recently constructed viscometer. A consideration of the effects of possible random and systematic errors led to an estimated error in viscosity of  $\pm 0.7\%$ . This estimate was in accord with a check of accuracy in the form of measurements of the viscosity of pure carbon dioxide relative to that of argon.

Measurements of the pure-component viscosities and those of argon + carbon dioxide mixtures required corrections for the kinetic energy gained by the flowing gas, for gas imperfection, and for slip flow due to nonzero velocity at the walls of the capillary tube. A correction also arises from the increased resistance to flow due to the curvature of the helical capillary tube. This resistance is measured in terms of a dimensionless quantity call the Dean number, which increases with flow rate. Only runs with Dean numbers less than 6 were used. Under these conditions the correction for curved pipe flow is negligible.

Once the flow times are corrected as described, then it may be shown that for a particular temperature the viscosities  $\eta$  are related to the corrected flow times  $t^{\text{corr}}$  by the simple working equation

$$\frac{\eta_{\text{sample}}}{\eta_{\text{argon}}} = \frac{t_{\text{sample}}^{\text{corr}}}{t_{\text{argon}}^{\text{corr}}} \quad (3)$$

The corrected flow time ratios, i.e., the right-hand terms of Eq. (3), are shown in Tables I and II. The second columns show the mean pressure in the capillary tube during a particular run. Those measurements carried out with the newer apparatus are indicated with a superscript asterisk. To convert the relative viscosities to absolute values, argon standard viscosities must be used to supply the term  $\eta_{\text{argon}}$ . They were in the form

$$\ln(\eta_{\text{argon}}/\eta_0) = A \ln(T/T_0) + B/T + C/T^2 + D \quad (4)$$

where  $T_0$  is 1 K, with coefficients as listed in Table III [13].

Also shown in Table III are the coefficients which provide the carbon dioxide viscosities recommended by Maitland and Smith [13]. All the carbon dioxide viscosities measured in this work were within  $\pm 0.5\%$  of these values.

Table I. Measured Viscosities of Pure CO<sub>2</sub> and 75% CO<sub>2</sub> + 25% Ar

| Temperature<br>(K) | Mean<br>pressure<br>(mm Hg) | Pure CO <sub>2</sub><br>flow time<br>ratio | Pure CO <sub>2</sub><br>viscosity<br>(10 <sup>-6</sup> kg·m <sup>-1</sup> ·s <sup>-1</sup> ) | Pure CO <sub>2</sub> mean<br>viscosity<br>(10 <sup>-6</sup> kg·m <sup>-1</sup> ·s <sup>-1</sup> ) | 75% CO <sub>2</sub><br>flow time<br>ratio | 75% CO <sub>2</sub><br>viscosity<br>(10 <sup>-6</sup> kg·m <sup>-1</sup> ·s <sup>-1</sup> ) | 75% CO <sub>2</sub> mean<br>viscosity<br>(10 <sup>-6</sup> kg·m <sup>-1</sup> ·s <sup>-1</sup> ) |
|--------------------|-----------------------------|--|--|---|---|---|--|
| 213.15             | 156.9                       | 0.79212                                    | 10.768   | 10.77   | 0.89723                                   | 12.197  | 12.20  |
|                    | 154.6                       | 0.79089                                    | 10.752   |   | 0.89699                                   | 12.194  |  |
| 213.15*            | 153.4                       | 0.79292                                    | 10.779   |   | 0.89748                                   | 12.201  |  |
|                    | 152.5                       | 0.78788                                    | 10.711   | 10.73   | 0.89493                                   | 12.166  | 12.18  |
|                    | 148.6                       | 0.78862                                    | 10.721   |   | 0.89498                                   | 12.167  |  |
|                    | 145.1                       | 0.79032                                    | 10.744   |   | 0.89706                                   | 12.195  |  |
| 233.15             | 156.9                       | 0.80155                                    | 11.734   | 11.74   | 0.91052                                   | 13.329  | 13.33  |
|                    | 154.6                       | 0.80142                                    | 11.732   |   | 0.91033                                   | 13.326  |  |
| 253.15             | 153.4                       | 0.80181                                    | 11.738   |   | 0.91118                                   | 13.339  |  |
|                    | 156.9                       | 0.81751                                    | 12.790   | 12.79   | 0.92072                                   | 14.405  | 14.40  |
|                    | 154.6                       | 0.81844                                    | 12.805   |   | 0.92143                                   | 14.416  |  |
|                    | 153.4                       | 0.81721                                    | 12.785   |   | 0.91990                                   | 14.392  |  |
| 301.00             | 233.7                       | 0.83736                                    | 15.003   | 15.02   | 0.94255                                   | 16.888  | 16.90  |
|                    | 231.4                       | 0.83863                                    | 15.026   |   | 0.94320                                   | 16.900  |  |
| 313.15             | 229.7                       | 0.83935                                    | 15.039   |   | 0.94401                                   | 16.914  |  |
|                    | 310.7                       | 0.84558                                    | 15.616   | 15.62   | 0.94985                                   | 17.542  | 17.54  |
|                    | 308.2                       | 0.84593                                    | 15.623   |   | 0.95008                                   | 17.546  |  |
|                    | 306.5                       | 0.84593                                    | 15.620   |   | 0.94977                                   | 17.540  |  |
| 353.15             | 310.7                       | 0.86511                                    | 17.492   | 17.48   | 0.96753                                   | 19.563  | 19.58  |
|                    | 308.2                       | 0.86455                                    | 17.481   |   | 0.96807                                   | 19.574  |  |
|                    | 306.5                       | 0.86421                                    | 17.474   |   | 0.96904                                   | 19.593  |  |

Table II. Measured Viscosities of 50% CO<sub>2</sub> + 50% Ar and 25% CO<sub>2</sub> + 75% Ar

| Temperature<br>(K) | Mean<br>pressure<br>(mm Hg) | 50% CO <sub>2</sub><br>flow time<br>ratio | 50% CO <sub>2</sub><br>viscosity<br>(10 <sup>-6</sup> kg · m <sup>-1</sup> · s <sup>-1</sup> ) | 50% CO <sub>2</sub> mean<br>viscosity<br>(10 <sup>-6</sup> kg · m <sup>-1</sup> · s <sup>-1</sup> ) | 25% CO <sub>2</sub><br>flow time<br>ratio | 25% CO <sub>2</sub><br>viscosity<br>(10 <sup>-6</sup> kg · m <sup>-1</sup> · s <sup>-1</sup> ) | 25% CO <sub>2</sub> mean<br>viscosity<br>(10 <sup>-6</sup> kg · m <sup>-1</sup> · s <sup>-1</sup> ) |
|--------------------|-----------------------------|---|--|---|---|--|---|
| 213.15             | 156.9                       | 1.0074                                    | 13.695   | 13.69   | 1.1244                                    | 15.285   | 15.31   |
|                    | 154.6                       | 1.0064                                    | 13.681   |   | 1.1256                                    | 15.302   |   |
|                    | 153.4                       | 1.0067                                    | 13.685   |   | 1.1282                                    | 15.337   |   |
| 213.15*            | 152.5                       | 1.0042                                    | 13.652   | 13.68   | 1.1184                                    | 15.204   | 15.22   |
|                    | 148.6                       | 1.0062                                    | 13.679   |   | 1.1199                                    | 15.224   |   |
|                    | 145.1                       | 1.0091                                    | 13.718   |   | 1.1205                                    | 15.232   |   |
| 233.15             | 156.9                       | 1.0184                                    | 14.909   | 14.93   | 1.1326                                    | 16.580   | 16.60   |
|                    | 154.6                       | 1.0200                                    | 14.932   |   | 1.1346                                    | 16.609   |   |
|                    | 153.4                       | 1.0202                                    | 14.935   |   | 1.1343                                    | 16.605   |   |
| 253.15             | 156.9                       | 1.0268                                    | 16.065   | 16.07   | 1.1433                                    | 17.887   | 17.88   |
|                    | 154.6                       | 1.0281                                    | 16.085   |   | 1.1436                                    | 17.892   |   |
|                    | 153.4                       | 1.0265                                    | 16.060   |   | 1.1410                                    | 17.851   |   |
| 301.00             | 156.9                       | 1.0497                                    | 18.808   | 18.83   | 1.1571                                    | 20.733   | 20.75   |
|                    | 154.6                       | 1.0504                                    | 18.802   |   | 1.1594                                    | 20.773   |   |
|                    | 153.4                       | 1.0529                                    | 18.865   |   | 1.1584                                    | 20.755   |   |
| 313.15             | 310.7                       | 1.0571                                    | 19.522   | 19.54   | 1.1672                                    | 21.557   | 21.58   |
|                    | 308.2                       | 1.0589                                    | 19.555   |   | 1.1691                                    | 21.591   |   |
|                    | 306.5                       | 1.0581                                    | 19.541   |   | 1.1685                                    | 21.580   |   |
| 353.15             | 310.7                       | 1.0725                                    | 21.686   | 21.69   | 1.1805                                    | 23.869   | 23.86   |
|                    | 308.2                       | 1.0722                                    | 21.679   |   | 1.1782                                    | 23.823   |   |
|                    | 306.5                       | 1.0734                                    | 21.704   |   | 1.1811                                    | 23.881   |   |

**Table III.** Coefficients of Eq. (3) for the Argon and Carbon Dioxide Standard Viscosities, for the Combined Interaction Viscosities  $\eta_{12}$  and Diffusion Coefficients  $D_{12}$ 

|                 | <i>A</i> | <i>B</i><br>(K) | <i>C</i><br>(K <sup>2</sup> ) | <i>D</i> | $\frac{\eta_0}{(10^{-6} \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1})}$<br>or $pD_{12,0}(\text{N} \cdot \text{s}^{-1})^a$ |
|-----------------|----------|-----------------|-------------------------------|----------|---|
| CO <sub>2</sub> | 0.52662  | -97.589         | 2650.7                        | -2.6892  | 20.32   |
| Ar              | 0.59077  | -92.577         | 2990.4                        | -3.0755  | 22.28   |
| $\eta_{12}$     | 0.35058  | -252.56         | 14196.0                       | 1.6409   | 0.1   |
| $D_{12}$        | 1.42828  | -206.035        | 11548.0                       | -9.48328 | 0.1   |

<sup>a</sup>  $p$  = pressure, N · m<sup>-2</sup>.

Interaction viscosities  $\eta_{12}$  were then calculated from the mixture viscosities and the viscosities of the pure components. The omega integral ratios  $A_{12}^*$  required in these calculations were those of the argon BBMS potential function [14]. The well depth  $\epsilon/k$  of 182 K used to calculate the reduced temperatures was that of Hobley et al. [15]. The resulting  $T^*$  and  $A_{12}^*$  values are shown in Table IV. Also shown are the calculated interaction viscosities at each temperature, calculated from the viscosities of the pure gases and those of the various mixtures shown at the head of each column. If both experiment and theory were perfectly accurate, identical  $\eta_{12}$  values for each mixture at a particular temperature would be obtained. In practice, the calculation of interaction viscosity is sensitive to errors in the pure component and mixture viscosities, and consequently the calculated  $\eta_{12}$  values are estimated to be accurate to  $\pm 1.4\%$ .

The interaction viscosities were combined with those of Hobley et al. [15] over the temperature range 301 to 521 K. A best-fitting curve of the

**Table IV.** Interaction Viscosities Calculated from the Experimental Mixture Viscosities of this Work

| Temperature<br>(K) | $T^*$ | $A_{12}^*$ | $\eta_{12}(10^{-6} \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1})$ |                              |                              |
|--------------------|-------|------------|---|------------------------------|------------------------------|
|                    |       |            | 50% Ar + 50% CO <sub>2</sub>  | 25% Ar + 75% CO <sub>2</sub> | 75% Ar + 25% CO <sub>2</sub> |
| 213.15             | 1.17  | 1.1084     | 14.104  | 14.117                       | 14.283                       |
| 213.15*            | 1.17  | 1.1084     | 14.096  | 14.127                       | 13.996                       |
| 233.15             | 1.28  | 1.1073     | 15.380  | 15.503                       | 15.340                       |
| 253.15             | 1.39  | 1.1062     | 16.348  | 16.531                       | 16.581                       |
| 301.00             | 1.65  | 1.1035     | 19.400  | 19.414                       | 19.291                       |
| 313.15             | 1.72  | 1.1029     | 20.134  | 20.108                       | 20.181                       |
| 353.15             | 1.94  | 1.1008     | 22.313  | 22.374                       | 22.313                       |

form of Eq. (4) was calculated for the combined data set, resulting in the coefficients listed in Table III. The deviations of the present interaction viscosities and those of other workers from this curve are shown in Fig. 2. The data are generally within  $\pm 2\%$  of the curve, with the data of Kestin and Ro [16] exhibiting the most positive deviation.

Ar + CO<sub>2</sub> binary diffusion coefficients were calculated by means of the first-order relation [10]

$$D_{12} = \frac{3}{5} \frac{\eta_{12} A_{12}^* RT}{P m_{12}} \quad (5)$$

where  $P$  is the pressure and  $R$  the gas constant. Table III gives the coefficients for the best-fitting curve for these data combined with those of Hobley et al. Figure 3 shows that the diffusion coefficients calculated in this work lie within 1.2% of the fitting curve, within the expected experimental error of the interaction viscosities from which they are derived. The diffusion coefficients from this work and from Hobley et al. are calculated for each separate mixture, the latter having a relatively wide scatter. The experimental interaction viscosities of Kestin and Ro [16] give diffusion coefficients which are also very close to the fitted curve, deviating by not more than 0.5%. However Kestin and Ro's corresponding-states model [17] deviates from the fitting curve by more than 1%.

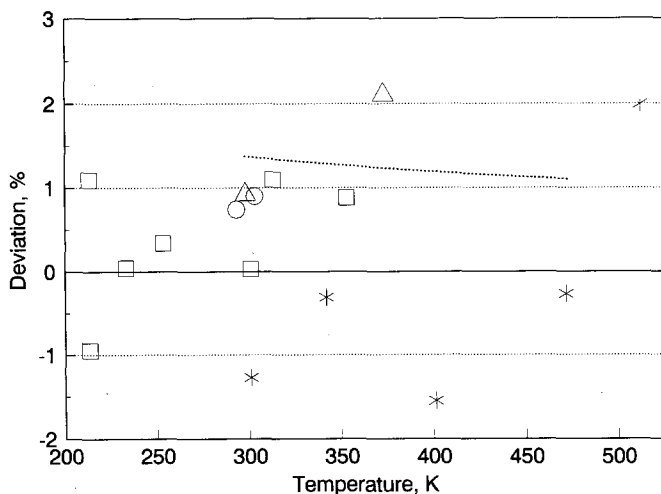
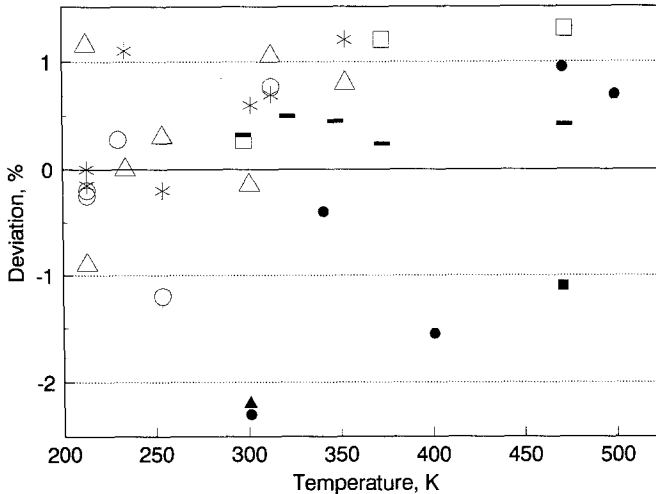


Fig. 2. Deviations of interaction viscosities  $\eta_{12}$  from the fitting curve of Table III. □, This work; \*, Hobley et al. [15]; Δ, Kestin and Ro [16]; ·····, Kestin and Ro [17]; ○, Kestin et al. [18].





**Fig. 3.** Deviations of calculated binary diffusion coefficients  $D_{12}$  from the fitting curve of Table III. \*, This work, 25% Ar (+75% CO<sub>2</sub>); O, this work, 50% Ar; Δ, this work, 75% Ar; ▲, Hobley et al., 25% Ar [15]; ●, Hobley et al., 50% Ar [15]; ■, Hobley et al., 75% Ar [15]; ■, Kestin and Ro [16]; □, Kestin and Ro [17].

### 3. ANISOTROPIC PAIR POTENTIAL ENERGY FUNCTIONS

In this section we describe the various potential energy functions proposed for the Ar–CO<sub>2</sub> interaction by other workers, then calculate viscosities from them via the Mason–Monchick approximation.

#### 3.1. The Potential Functions of Pack and Co-Workers

Pack and co-workers [1–3] have calculated various anisotropic potential energy surfaces  $U(r, \theta)$  for the argon + carbon dioxide interaction. In this work we apply the Mason–Monchick approximation to a representative selection of three, referred to as Pack 1, Pack 2, and Pack 3. The Pack 1 surface [1] was calculated by means of the electron gas approximation at short range, of the form

$$\begin{aligned} U(r, \theta) &= E(r, \theta) - E(\infty, \text{any } \theta) \\ &= U_{\text{HF}}(r, \theta) + U_{\text{COR}}(r, \theta) \end{aligned} \quad (6)$$

and

$$V_{\text{HF}} = V_{\text{COUL}} + V_{\text{KIN}} + V_{\text{EXC}} \quad (7)$$

where  $E$  is the absolute potential energy of the molecule, and the subscripts refer to the Hartree Fock, correlation, coulomb, kinetic, and exchange contributions to the interaction energy. At long range, the surface was joined to a semiempirical van der Waals potential, incorporating theoretical  $C_6$  and  $C_8$  coefficients [19]. The potential parameters were adjusted so that there was good agreement with experimental second virial coefficients of Cottrell et al. [20] and Brewer [21], and there was reasonable agreement with high-energy scattering data. Various ways of joining the short- and long-range functions were presented, and our Pack 1 surface is that which gives the highest angular dependence of the well depth  $\epsilon$ .

A second surface, Pack 2 [2], takes into account the rotational inelasticity in carbon dioxide. It gives good agreement with differential scattering cross sections. Again, various surfaces are described, and we employ the surface with highest angular dependence of  $\epsilon$ .

To facilitate study of the damping of rainbow oscillations and diffraction oscillations in differential scattering cross sections, Pack [3] parameterized the previous surfaces in terms of an angle-dependent Lennard-Jones 12-6 potential function of the form

$$U(r, \theta) = \epsilon(\theta) \{ [r_m(\theta)/r]^{12} - 2[r_m(\theta)/r]^6 \} \quad (8)$$

with

$$\epsilon(\theta) = \bar{\epsilon} [1 + aP_2(\cos \theta)] \quad (9)$$

and

$$r_m(\theta) = \bar{r}_m [1 + bP_2(\cos \theta)] \quad (10)$$

$P_2$  being the second Legendre polynomial. For Ar + CO<sub>2</sub>, Pack used  $\bar{\epsilon} = 200$  K and  $\bar{r}_m = 0.388$  nm. Surfaces with various  $a$  and  $b$  values were presented, of which our Pack 3 surface has  $a = 0.0$  and  $b = 0.2$ .

We have determined the Mason-Monchick angular averages of the Pack 1, 2, and 3 surfaces. The calculations used the seven positive pivots of a 14-point Gauss Legendre quadrature which yields collision integrals  $\Omega_{12}^{(2,2)*}$  with an estimated uncertainty of no more than  $\pm 0.2\%$ . Figure 4 shows cross sections of the Pack 1, Pack 2, and Pack 3 potential energy surfaces at two of the seven pivots used in our calculations, these being at  $\theta = 88.5$  and  $13.0^\circ$ . Interaction viscosities derived from the collision integrals are listed in Table V.

The interaction viscosities calculated from the Pack 1 surface are within 0.25% of similar calculations by Maitland et al. [22] as modified by Vesovic [23]. The corresponding diffusion coefficients  $D_{12}$  of Maitland

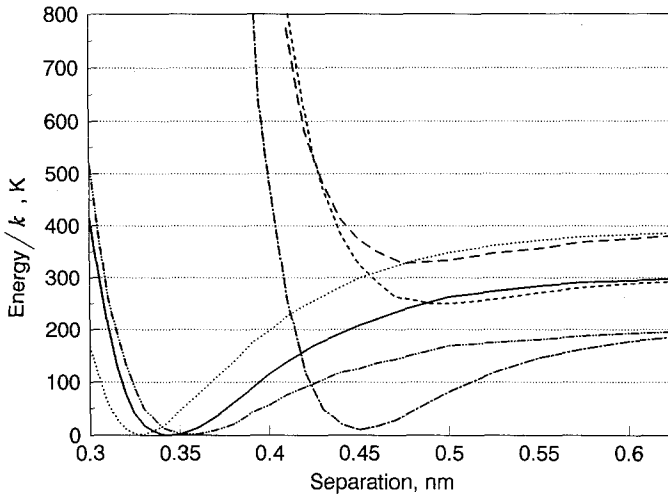


Fig. 4. Cross sections through potential energy surfaces of Pack and co-workers at  $\theta = 88.5$  and  $13^\circ$ . Pack 1: —,  $88.5^\circ$ ; - - - - ,  $13^\circ$  [1]. Pack 2: . . . . .,  $88.5^\circ$ ; - - - ,  $13^\circ$  [2]. Pack 3: - · - · - ,  $88.5^\circ$ ; - · - · - ,  $13^\circ$  [3].

et al. agree well with the experimentally based fitting curve of Table III, deviating by no more than 0.6%.

The deviation of the calculated viscosities about a fitted curve of experimental viscosities of this work and that of Hobley et al. [15] is shown in Fig. 5. The Pack 2 potential energy surface generates viscosities within 3% of the experimentally based interaction viscosities, whereas Pack 1 and Pack 3 generate viscosities up to 6% too low.

Table V. Interaction Viscosities  $\eta_{12}$  Calculated from the Potential Energy Surfaces of Pack and Co-Workers [1-3]

| Temperature<br>(K) | Pack 1 [1]<br>( $10^{-6} \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$ ) | Pack 2 [2]<br>( $10^{-6} \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$ ) | Pack 3 [3]<br>( $10^{-6} \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$ ) |
|--------------------|--|--|--|
| 200.0              | 13.30  | 13.25  | 12.66  |
| 300.0              | 18.82  | 18.96  | 18.35  |
| 400.0              | 23.67  | 24.14  | 23.48  |
| 500.0              | 28.09  | 28.86  | 28.12  |
| 600.0              | 32.12  | 33.20  | 32.36  |
| 700.0              | 35.87  | 37.24  | 36.30  |
| 800.0              | 39.41  | 41.06  | 39.99  |
| 900.0              | 42.78  | 44.70  | 43.49  |
| 1000.0             | 46.03  | 48.19  | 46.83  |

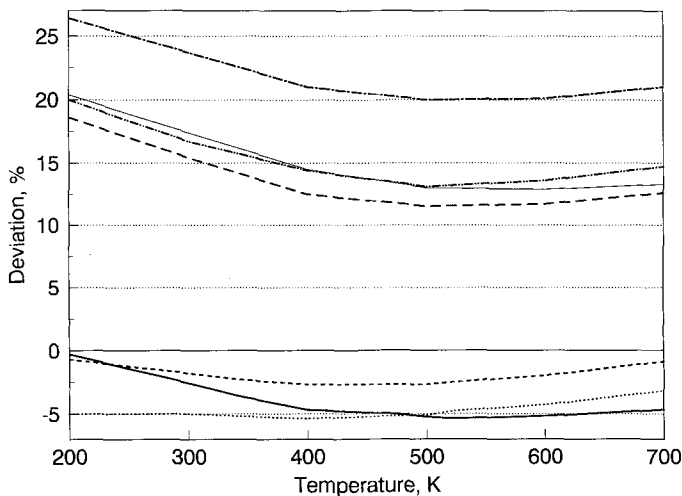


Fig. 5. Deviations of calculated interaction viscosities from the experimental fitting curve of Table III. —, Pack 1 [1]; ----, Pack 2 [2]; ·····, Pack 3 [3]. Hough and Howard [4, 5]: - · - ·, A3; - - - -, A3-M; - · - ·, AD; —, AD-M.

### 3.2. The Potential Functions of Hough and Howard

The latest angle-dependent potential energy surfaces for the Ar + CO<sub>2</sub> system were reported by Hough and Howard [4, 5]. The potentials of Hough and Howard were obtained by a simultaneous least-squares fit to molecular-beam electric resonance spectra, mixed second virial coefficients, and mean square torque measurements. Their potential function was expressed as an angle-dependent parameterisation of the Maitland–Smith  $n(r^*)-6$  function:

$$U(r, \theta) = [\varepsilon(\theta)/\{n(\theta) - 6\}] [6x(\theta)^{-n(\theta)} - n(\theta)x(\theta)^{-6}] \quad (11)$$

with

$$n(\theta) = m(\theta) + \gamma[x(\theta) - 1] \quad (12)$$

and

$$x(\theta) = r/r_m(\theta) \quad (13)$$

Here  $m(\theta)$  is related to the radial curvature at the radial minimum and is a shape skewing parameter. For noble gases a value of  $m = 13$  is usually applied, though this parameter has been varied in two of the Hough and

Howard potentials. The value of  $\gamma$  is less crucial, and they used a value of 9 in all their proposed potential functions. The parameters  $\varepsilon(\theta)$  and  $r_m(\theta)$  were expressed as sums of even Legendre polynomials, in a similar style to the Pack 3 potential surface parameterisation.

Figure 6 shows cross sections through the 3A potential energy surface for the three angle parameterization with the value of  $m$  fixed at 13 and the potential of the linear geometry fixed with  $r_m(\theta)$  at between 0.49 and 0.50 nm. The variation of  $r_m(\theta)$  is relatively similar to those of Pack, but the variation in well depths is some way between the two extremes of the Pack 2 and 3 potentials, having well depths between 201 and 56 K.

The second potential energy surface described by Hough and Howard was the 3A-M, which was the 3A potential with the repulsive parameter  $m$  included in the optimization. This gives a potential surface which is very similar to 3A, having slightly shallower well depths as  $\theta$  approaches zero. However, the main difference is in the linear configuration, due to the fact that  $r_m$  is not fixed.

The third potential surface described is the angle-dependent AD potential, with  $m$  fixed at 13. A cross section at  $\theta = 13^\circ$  is shown in Fig. 6. A problem pointed out by Hough and Howard for this surface was that the Legendre polynomial  $P_4$  dominates the  $P_2$  term near  $90^\circ$ . This causes the radial distance to decrease on either side of the potential minimum, which is unrealistic. The final potential quoted by Hough and Howard was the

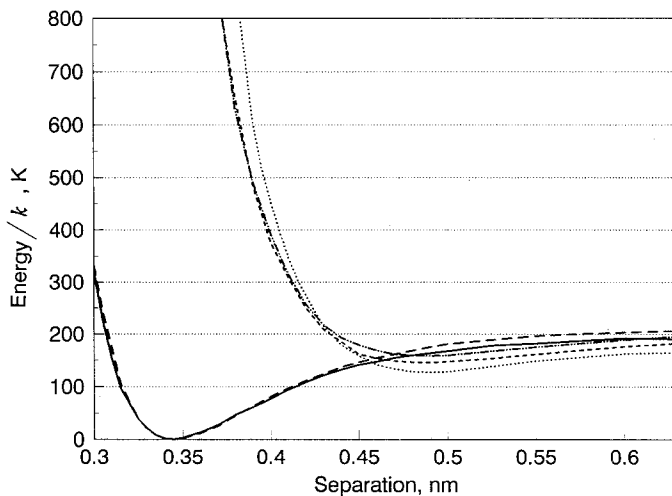


Fig. 6. Cross sections through potential energy surfaces of Hough and Howard [4, 5] at  $\theta = 88.5$  and  $13^\circ$ . —, 3A,  $88.5^\circ$ ; - - - -, 3A,  $13.0^\circ$ ; ·····, AD,  $13.0^\circ$ ; - - - -, AD-M,  $88.5^\circ$ ; - · - · - ·, AD-M,  $13.0^\circ$ .

**Table VI.** Interaction Viscosities  $\eta_{12}$  Calculated from the Potential Energy Surfaces of Hough and Howard [4, 5]

| Temperature<br>(K) | 3A<br>( $10^{-6} \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$ ) | 3A-M<br>( $10^{-6} \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$ ) | AD<br>( $10^{-6} \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$ ) | AD-M<br>( $10^{-6} \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$ ) |
|--------------------|--|--|--|--|
| 200.0              | 15.85  | 16.03  | 16.88  | 16.10  |
| 300.0              | 22.33  | 22.59  | 23.91  | 22.72  |
| 400.0              | 28.00  | 28.38  | 30.07  | 28.47  |
| 500.0              | 33.13  | 33.65  | 35.61  | 33.60  |
| 600.0              | 37.89  | 38.55  | 40.73  | 38.30  |
| 700.0              | 42.36  | 43.16  | 45.52  | 42.68  |
| 800.0              | 46.62  | 47.55  | 50.08  | 46.82  |
| 900.0              | 50.67  | 51.74  | 54.45  | 50.76  |
| 1000.0             | 54.62  | 55.77  | 58.65  | 54.56  |

AD-M, which is the same as the AD surface with  $m$  included as a potential variable (Fig. 6). The AD and AD-M cross sections at  $88.5^\circ$  are similar on the scale of Fig. 6.

Table VI lists the viscosity values calculated from the Hough and Howard potentials using the Mason–Monchick approximation. They are up to 27% higher than the experimentally based interaction viscosities, as shown in Fig. 5.

#### 4. DISCUSSION

The calculation of interaction viscosities from potential energy surfaces by means of the Mason–Monchick approximation, and subsequent comparison with experimentally based values, has been shown to provide a powerful test of the accuracy of the surfaces. The Pack 2 potential energy surface [2] reproduces the interaction viscosities with reasonable accuracy. This surface is Pack's own optimization of the Pack 1 surface, whereas the Pack 3 is merely a trial parameterization. The optimum fit of the Pack 2 surface therefore agrees with Pack's own findings based on entirely different experimental properties. The surfaces of Howard and co-workers give very much greater deviations. The Mason–Monchick approximation is considered accurate to a few percent, and the errors must arise from the quality of the potential energy surfaces. The discrepancies may occur because of the use of insufficiently flexible parameterization or from errors in the data on which the potentials are based.

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